

Number with different bases

Decimal base 10	Binary base 2	Octal base 8	Hexadecimal base 16
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	10 - A
11	1011	13	11 - B
12	1100	14	12 - C
13	1101	15	13 - D
14	1110	16	14 - E
15	1111	17	15 - F

Base (or) radix

In general a number expressed in base r system has co-efficients multiplied by powers of r:

$$a_n r^n + a_{n-1} r^{n-1} + \dots + a_2 r^2 + a_1 r^1 + a_0 r^0 + a_{-1} r^{-1} + a_{-2} r^{-2} + \dots + a_{-n} r^{-n}$$

The coefficients of range 0 in value from 0 to r-1.

- EX. decimal (30)<sub>10</sub>      binary (1011)<sub>2</sub>      octal (23)<sub>8</sub>  
 Hexadecimal (5A)<sub>16</sub>      base 5 (4021)<sub>5</sub>

① Convert 41 into binary & octal & Hexadecimal

$$\begin{array}{r}
 2 \overline{) 41} \\
 \underline{20} \phantom{0} \\
 2 \overline{) 20} \\
 \underline{10} \phantom{0} \\
 2 \overline{) 10} \\
 \underline{5} \phantom{0} \\
 2 \overline{) 5} \\
 \underline{2} \phantom{0} \\
 2 \overline{) 2} \\
 \underline{1} \phantom{0} \\
 1 \phantom{0}
 \end{array}$$

$$(41)_{10} = (101001)_2$$

$$= \underline{101} \underline{001}$$

$$= 5 \ 1 = (51)_8$$

$$8 \overline{) 41} = (51)_8$$

$$5 \phantom{0} \phantom{0} \phantom{0}$$

$$16 \overline{) 41} = (29)_{16} = 29_H$$

$$2 \phantom{0} \phantom{0} \phantom{0} \phantom{0}$$

$$= \underline{101} \underline{001} = (29)_{16}$$

② Convert  $(0.6875)_{10}$  to binary

$$0.6875 \times 2 = 1.3750 = 1$$

$$0.3750 \times 2 = 0.7500 = 0$$

$$0.75 \times 2 = 1.50 = 1$$

$$0.50 \times 2 = 1.00 = 1$$

$$(0.6875)_{10} = (0.1011)_2$$

③ Convert  $(0.513)_{10}$  to octal

$$0.513 \times 8 = 4.104$$

$$0.104 \times 8 = 0.832$$

$$0.832 \times 8 = 6.656$$

$$0.656 \times 8 = 5.248$$

$$0.248 \times 8 = 1.984$$

$$0.984 \times 8 = 7.872$$

$$(0.513)_{10} = (0.406517)_8$$

④  $(41.6875)_{10}$  ——— fractional separate.

decimal separate +

$$(41.6875)_{10} = (101001.1011)_2$$

⑤  $(101001.1011)_2$  convert to decimal

$$\begin{array}{r}
 101001 \\
 \hline
 1 \times 2^0 = 1 \times 1 = 1 \\
 0 \times 2^1 = 0 \times 2 = 0 \\
 0 \times 2^2 = 0 \times 4 = 0 \\
 1 \times 2^3 = 1 \times 8 = 8 \\
 0 \times 2^4 = 0 \times 16 = 0 \\
 1 \times 2^5 = 1 \times 32 = 32 \\
 \hline
 41
 \end{array}$$

$$\begin{array}{r}
 0.1011 \\
 \hline
 1 \times 2^{-1} = 1 \times 0.5 = 0.5 \\
 0 \times 2^{-2} = 0 \times 0.25 = 0.0 \\
 1 \times 2^{-3} = 1 \times 0.125 = 0.125 \\
 1 \times 2^{-4} = 1 \times 0.0625 = 0.0625 \\
 \hline
 = 0.6875
 \end{array}$$

Binary Arithmetic

A B sum carry

$$00 \quad 0 \quad 0$$

$$01 \quad 1 \quad 0$$

$$10 \quad 1 \quad 0$$

$$11 \quad 0 \quad 1$$

① Add 28 & 15 in binary

$$\begin{array}{r}
 {}^2 \sqrt{28} \\
 {}^2 \sqrt{14} - 0 \\
 {}^2 \sqrt{7} - 0 \\
 {}^2 \sqrt{3} - 1 \\
 \underline{1 - 1}
 \end{array}
 \qquad
 \begin{array}{r}
 {}^2 \sqrt{15} \\
 {}^2 \sqrt{7} - 1 \\
 {}^2 \sqrt{3} - 1 \\
 \underline{1 - 1}
 \end{array}
 \qquad
 \begin{array}{r}
 (28)_{10} = 11100 \\
 (15)_{10} = 1111 \\
 \hline
 101011
 \end{array}$$

② Binary subtraction

Rules	A	B	Diff	Borrow
	0	0	0	0
	0	1	1	1
	1	0	1	0
	1	1	0	0

subtract  $(0101)_2$  from  $(1011)_2$

$$\begin{array}{r}
 1011 \quad \text{--- Decimal } 11 \\
 - 0101 \quad \text{--- Decimal } 5 \\
 \hline
 0110 \quad \text{--- Decimal } 6
 \end{array}$$

Binary Multiplication

Rules :

- $0 \times 0 = 0$
- $0 \times 1 = 0$
- $1 \times 0 = 0$
- $1 \times 1 = 01$

Ex.  $011 \times 110$

$$\begin{array}{r}
 011 \\
 \times 110 \\
 \hline
 000 \\
 011 \\
 10010 \\
 \hline
 10010
 \end{array}$$

Ex:2

$$\begin{array}{r}
 1110 \times 1010 \\
 \hline
 10001100
 \end{array}$$

# Binary division

Rules

$$0 \div 1 = 0$$

$$1 \div 1 = 1.$$

Ex. 1  $\Rightarrow$  Divide  $(11011011)_2$  by  $(110)_2$

$$\begin{array}{r} 100100 \\ 110 \overline{) 11011011} \\ \underline{110} \phantom{000} \\ 000110 \phantom{00} \\ \underline{110} \phantom{00} \\ 100011 \phantom{0} \\ \underline{100011} \\ 0 \phantom{00} \end{array}$$

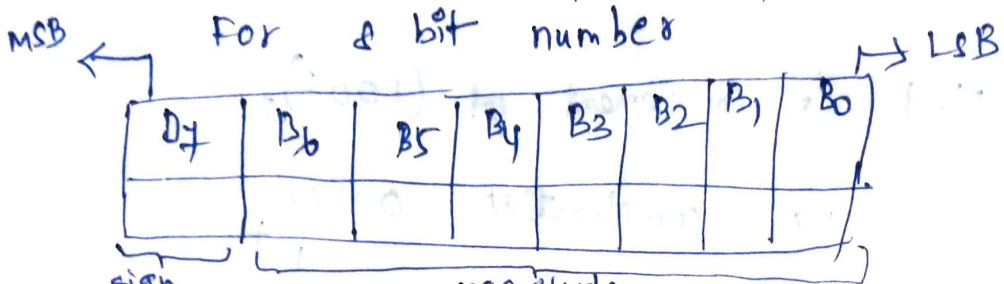
Ex 2: Divide  $(110101101)_2$  by  $(101)_2$

$$\begin{array}{r} 1010101 \\ 101 \overline{) 110101101} \\ \underline{101} \phantom{00000} \\ 0110 \phantom{000} \\ \underline{101} \phantom{000} \\ 0111 \phantom{00} \\ \underline{101} \phantom{00} \\ 01001 \phantom{0} \\ \underline{101} \phantom{00} \\ 100 \phantom{0} \\ \underline{100} \\ 0 \phantom{00} \end{array}$$

# Signed Binary numbers

Unsigned numbers represent only magnitude

Signed numbers  $\Rightarrow$  MSB represents the sign  
 $MSB=0 \Rightarrow +ve$  number &  $MSB=1 \Rightarrow -ve$  number



+6 =	0	0	0	0	0	1	0
-14 =	1	0	0	0	1	1	0
+24 =	0	0	0	1	1	0	0
-64 =	1	1	0	0	0	0	0

8 bit binary numbers decimal range 0 to 255  
In case signed 8 bit binary " " +127 to -128

Maximum positive number = +127 = 0111 1111

Maximum negative number = -128 = 1111 1111

-128 represented as 1000 0000 (2)

## 1's complement representation

change all 1's to zero & All 0's to 1.

(1) Find 1's complement of  $(1101)_2 \Rightarrow (0010)_2$

(2)  $(101100)_2 \Rightarrow (0100110)_2$

2's Complement Representation

2's complement = 1's complement + 1

2's complement form is used to represent -ve no

① Find 2's complement of  $(1001)_2$ .

$$\begin{array}{r}
 \text{1's complement} \quad 0110 \\
 \phantom{0110} \phantom{00} \phantom{00} \phantom{00} + \\
 \hline
 \phantom{0110} \phantom{00} \phantom{00} \phantom{00} 1 \\
 \hline
 0111 \quad \text{2's complement}
 \end{array}$$

② Find 2's complement of  $(10100011)_2$ .

$$\begin{array}{r}
 10100011 \quad \text{number} \\
 01011100 \quad \text{1's complement} \\
 + \\
 \hline
 01011101 \Rightarrow \text{2's complement}
 \end{array}$$

1's Complement subtraction

(i) Subtraction of smaller Number from larger number

Method

1. Determine the 1's complement of smallest no.
2. Add the 1's complement to the larger no
3. Remove the carry & add it to the result

This is called end-around carry

EX (1)

subtract  $001011_2$  from  $111001_2$

$$\begin{array}{r}
 111001 \\
 -001011 \\
 \hline
 110010 \quad (+) \\
 010100 \quad \text{1's comp} \\
 \hline
 1001101 \\
 \hline
 001110 \quad \text{Final answer}
 \end{array}$$

end around carry ←

## Subtraction of Larger Number from Small number

1. Determine the 1's complement of larger number.
2. Add the 1's complement of to the smaller number.
3. Answer in 1's complement form. To get the answer in true form take the 1's complement of assign negative sign to the answer.

EX Subtract  $111001_2$  from  $101011_2$  using 1's complement method.

$$\begin{array}{r} 101011 \\ \text{1's complement of } 111001: 000110 \end{array}$$

$$\begin{array}{r} 101011 \\ + 000110 \\ \hline 110001 \leftarrow \text{Answer in 1's comp form} \\ - 001110 \leftarrow \text{true Ans (after 1's comp)} \\ \hline \end{array}$$

## Advantages of 1's complement subtraction

- This is accomplished with an binary adder
- $\therefore$  This is useful in arithmetic logic circuits
- 1's complement of a number is easily obtained by inverting each bit in the number.

## 2's complement subtraction

① Subtraction of smaller Number from larger Number

Method 1. Determine the 2's complement of a smaller number



2. Add the 2's complement to the larger number
3. Discard the carry.

Ex subtract  $(101011)_2$  from  $(111001)_2$  using 2's complement method.

$$\begin{array}{r}
 \text{no: } 111001 \\
 \text{2's comp: } 010101 \\
 \hline
 \text{Discard } \textcircled{1} 001110 \\
 \hline
 001110 \text{ — Final answer.}
 \end{array}$$

Subtraction of Larger Number from smaller Number

Method:

1. Determine the 2's comp of larger no
2. Add the 2's comp to the smaller no
3. Answer in 2's comp form. To get the answer in the true form take the 2's complement; assign negative sign to the answer.

Ex subtract  $(111001)_2$  from  $(101011)_2$  using 2's complement method.

$$\begin{array}{r}
 \text{no: } 101011 \\
 \text{2's comp: } 000111 \\
 \hline
 110010 \text{ — Ans 2's comp form} \\
 \hline
 \underline{\underline{001110}} \text{ — Answer}
 \end{array}$$

$$\begin{array}{r}
 000110 \\
 \hline
 000111 \\
 \hline
 110010 \\
 \hline
 110001 \\
 \hline
 \underline{\underline{001110}} \text{ — 1's comp}
 \end{array}$$

# Gate level minimization

## Binary codes

The digital data is represented, stored & transmitted as groups of binary digits (bits). The group of bits, also known as binary code represented as both numbers & letters of the alphabets as well as many special characters & control functions.

## classification of Binary codes

### ① weighted codes:

Each digit position of the number represented a specific weight

Ex:- 567  $\Rightarrow$  weight of 5 is 100 & 6 is 10 & 7 is 1.

8421, 2421, 5211 are weighted codes.

### ② Non weighted codes

- are not assigned with any weight to each digit position.

Ex: Excess-3 & gray codes are non weighted code

### ③ Reflective codes

A code is said to be reflective when the code for 9's complement must be found. (Reflectivity)

2421, 5211 & excess 3 codes are reflective 8421 is not

### ④ Sequential codes:

- succeeding code is one binary number greater than

its preceding code.

401 & excess 3 are sequential.

### ⑤ Alphanumeric codes

Codes which contain both numbers & alphabetic characters are called alphanumeric codes.

Most commonly used codes are:

ASCII - American Standard Code for Information Interchange

EBCDIC - Extended Binary Coded Decimal Interchange code

Hollerith code.

### ⑥ Error detecting & correcting codes

When the digital information in the binary form is transmitted from one circuit or system to another

circuit or system an error may occur. This means a signal corresponding to 0 may change 1 or vice versa due to presence of noise. To maintain the

data Integrity between transmitter & Receiver

extra bit or more than one bit are added in the data. These extra bits allow the detection & some times correction of error in the data. The data along with the extra bit/bits forms the code.

- codes which allow only error detection are called error detecting codes & codes which allow error detection & correction are called error detecting & correcting code.

## Binary coded decimal (BCD) (8421)

- most common code 8421 BCD (4 bit)
- called 8421 BCD because the weights associated with 4 bits are 8421 from left to right.

Decimal digit	BCD code											
	8	4	2	1	2	4	2	1	8	4	2	1
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	0	0	0	1	0
3	0	0	1	1	0	0	1	1	0	0	1	1
4	0	1	0	0	0	1	0	0	0	1	0	0
5	0	1	0	1	1	0	1	1	0	1	0	1
6	0	1	1	0	1	1	0	0	0	1	1	0
7	0	1	1	1	1	1	0	1	1	0	0	0
8	1	0	0	0	1	1	1	0	1	0	0	1
9	1	0	0	1	1	1	1	1	1	0	1	0

## Excess-3 code

- Excess 3 code modified form of BCD number.
- The excess 3 code can be derived from the natural BCD code by adding 3 to each coded number.

Decimal digit	Excess 3		
0	0011	6	1001
1	0100	7	1010
2	0101	8	1011
3	0110	9	1100
4	0111		
5	1000		

In excess 3 code we get a's complement of a number by just complementing each bit. Due to this excess 3 code is called self complementing.

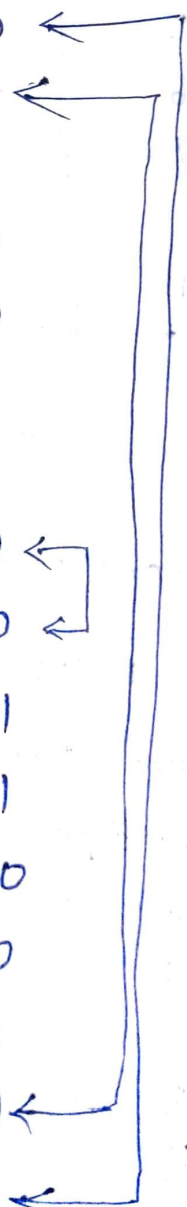
### Gray code

- Gray code is a special case of unit distance code.
  - In unit distance code bit patterns for two consecutive numbers differ in only one bit position.
- These codes are also called cyclic codes.

Decimal code

Gray code

0	0000
1	0001
2	0011
3	0010
4	0110
5	0111
6	0101
7	0100
8	1100
9	1101
10	1111
11	1110
12	1010
13	1011
14	1001
15	1000



Gray code any two adjacent code groups differ only in one bit position. The gray code is also called reflected code.

### Binary to Gray conversion

EX: Convert 10111011 in binary to equivalent gray code

Binary    1 → 0 → 1 → 1 → 1 → 0 → 1 → 1  
           ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓  
 Gray    1 1 1 0 0 1 1 0

### Gray to binary

convert gray code 101011 into its binary equivalent.

Gray code : 1 0 1 0 1 1  
               ↓ ↓ ↓ ↓ ↓ ↓  
 Binary    : 1 1 0 0 1 0

### Gate level minimization:

Usually literals (Boolean variables) & terms are arranged in one of the two standard forms of equations

- (i) Sum of Product form (SOP)
- (ii) Product of sum form (POS)

Sum of product :- <sup>(SOP)</sup>  
 (i)  $ABC + A\bar{B}\bar{C}$

(ii)  $\bar{P}Q + Q\bar{R} + PQR$

Product of sum : (POS) (i)  $(A+B)(B+\bar{C})$

(ii)  $(x+y)(y+\bar{z})(z+x)$

## Canonical Logic Forms

- Sum of products is Canonical (standard) <sup>sum</sup> products if every product term involves every literal or its complement.

$$SOP = AB + BC + ABC$$

Canonical (standard)  $SOP = ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC$

"  $POS = (A+B+C)(A+\bar{B}+C)$

Example : convert the given expression in canonical SOP.

$$Y = AC + AB + BC$$

$$= AC(B+\bar{B}) + AB(C+\bar{C}) + BC(A+\bar{A})$$

$$= ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC$$

$$= ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC$$

② Convert the given expression in canonical SOP form.

$$Y = A + AB + ABC$$

$$= A(B+\bar{B})(C+\bar{C}) + AB(C+\bar{C}) + ABC$$

$$= (A+B+\bar{A})(C+\bar{C}) + ABC + AB\bar{C} + ABC$$

$$= ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC + AB\bar{C}$$

$$= ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC$$

③ Convert the given expression in canonical POS form

$$Y = (A+B)(B+C)(A+C)$$

$$= (A+B+\bar{C})(B+C+\bar{A})(A+C+\bar{B})$$

$$= (A+B\bar{C})(A+B\bar{C})(A+B\bar{C})(A+B\bar{C})(A+B\bar{C})(A+B\bar{C})$$

$$= (A+B\bar{C})(A+B\bar{C})(A+B\bar{C})(A+B\bar{C})$$

$$\begin{aligned}
 \textcircled{4} \quad Y &= A(A+B)(A+B+C) \\
 &= (A+B\bar{B}+C\bar{C})(A+B+C)(A+B+C\bar{C}) \\
 &= \cancel{(A+B)(A+\bar{B})} + C\bar{C} \\
 &= (A+B\bar{B}+C)(A+B\bar{B}+\bar{C})(A+B+C) \\
 &\quad (A+B+C)(A+B\bar{C}) \\
 &= (A+B+C)(A+B+C)(A+B+\bar{C})(A+\bar{B}+\bar{C}) \\
 &\quad (A+B+C)(A+B+C)(A+B\bar{C}) \\
 &= (A+B+C)(A+\bar{B}+C)(A+B\bar{C})(A+\bar{B}+\bar{C})
 \end{aligned}$$

### Min terms & Max terms

Each individual term in canonical SOP form is called minterm & each individual term in canonical POS form is called maxterm.

For  $n$  variable logical function there are  $2^n$  minterms & equal number of maxterms. For 3 variables/literals  
 minterm = maxterm =  $2^3 = 8$ .

Variables			Min terms	Max terms
A	B	C	$m_i$	$M_i$
0	0	0	$\bar{A}\bar{B}\bar{C} = m_0$	$A+B+C = M_0$
0	0	1	$\bar{A}\bar{B}C = m_1$	$A+B+\bar{C} = M_1$
0	1	0	$\bar{A}B\bar{C} = m_2$	$A+\bar{B}+C = M_2$
0	1	1	$\bar{A}BC = m_3$	$A+\bar{B}+\bar{C} = M_3$
1	0	0	$A\bar{B}\bar{C} = m_4$	$\bar{A}+B+C = M_4$
1	0	1	$A\bar{B}C = m_5$	$\bar{A}+B+\bar{C} = M_5$
1	1	0	$AB\bar{C} = m_6$	$\bar{A}+\bar{B}+C = M_6$
1	1	1	$ABC = m_7$	$\bar{A}+\bar{B}+\bar{C} = M_7$



with these shorthand notations logical function can be represented as

$$1. Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C}$$

$$= m_0 + m_1 + m_3 + m_6 = \sum m(0, 1, 3, 6)$$

↗ denotes sum of products

$$2. Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$$

$$= M_1 + M_3 + M_6 = \prod M(1, 3, 6)$$

↗ Product of sum.

### Karnaugh Map Simplification

For simplification of boolean expressions by boolean algebra we need better understanding of boolean laws, rules & theorems.

- Time consuming process

map method gives us a systematic approach for simplifying a Boolean expression. The map method first proposed by veitch and modified by Karnaugh hence it is known as the veitch diagram or Karnaugh map. [Kmap]

2 Variables (4 cells)

	B	0	1
A	0	$m_0$	$m_1$
	1	$m_2$	$m_3$

3 Variables (8 cells)

	BC	00	01	11	10
A	0	$m_0$	$m_1$	$m_3$	$m_2$
	1	$m_4$	$m_5$	$m_7$	$m_6$

4 Variables (16 cells)

	CD	00	01	11	10
AB	00	$m_0$	$m_1$	$m_3$	$m_2$
	01	$m_4$	$m_5$	$m_7$	$m_6$
	11	$m_{12}$	$m_{13}$	$m_{15}$	$m_{14}$
	10	$m_8$	$m_9$	$m_{11}$	$m_{10}$

2 variable KMAP

$$f = \sum m(0, 2, 3) \xrightarrow{\text{SOP}}$$

①

	B	0	1
A	0	$\bar{A}\bar{B}$ <sub>0</sub>	$\bar{A}B$ <sub>1</sub>
	1	$AB$ <sub>2</sub>	$AB$ <sub>3</sub>

	B	0	1
A	0	1	0
	1	1	1

$$f = \bar{B} + A = A + \bar{B}$$

②

$$f = \bar{A}\bar{B} + \bar{A}B + AB \text{ using mapping.}$$

$$f = m_0 + m_1 + m_3 = \sum m(0, 1, 3)$$

	B	0	1
A	0	1	1
	1	0	1

$$f = \bar{A} + B$$



③

Mapping POS form.

$$f = (A+B)(\bar{A}+B)(\bar{A}+\bar{B})$$

$$f = \pi(0, 2, 3)$$

	B	0	1
A	0	0	0
	1	0	0

$$\bar{f} = \bar{B} + A \Rightarrow \bar{A} \cdot B$$

$$f = \overline{\bar{A} \cdot B}$$

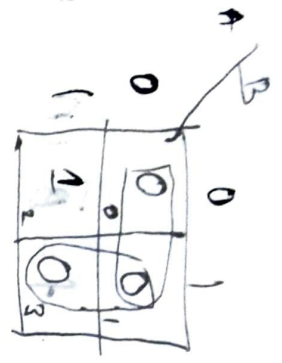
$$= \bar{A} \cdot \bar{B}$$

$$= \bar{A} \cdot \bar{B}$$



Reduce the following expression  $f = (A+B)(A+\bar{B})(A+\bar{B})(A+\bar{B})$

$= \prod (0, 1, 3)$



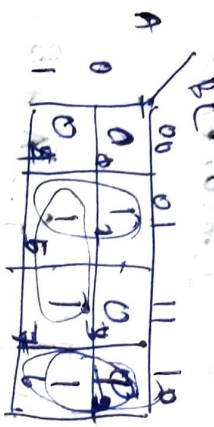
$\Rightarrow f = \bar{A} + B$

$f = A\bar{B}$

Three Variable KMap

Sum of products

Map the expression



$f = \bar{A}\bar{B}C + AB\bar{C} + \bar{A}B\bar{C}$

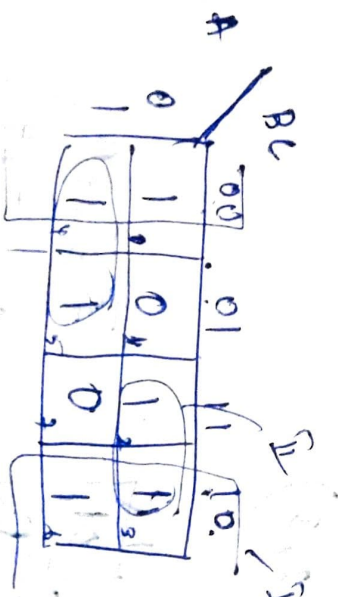
$f = \bar{B}C + AC + B\bar{C}$

(5)

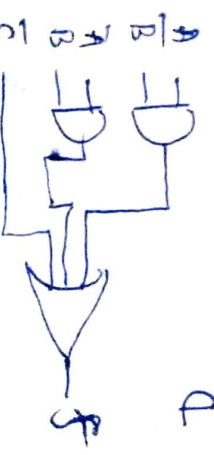
Reduce the expression  $f = \sum m(0,2,3,4,5,6)$  using Venn map

Implement it in AOI Logic.

$f = \sum m(0,2,3,4,5,6)$



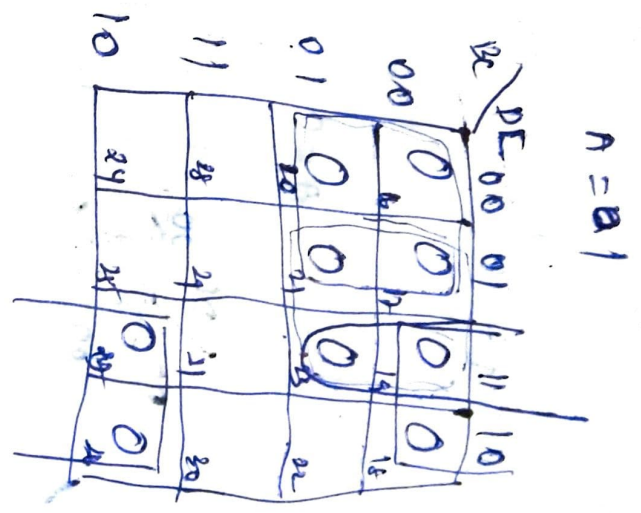
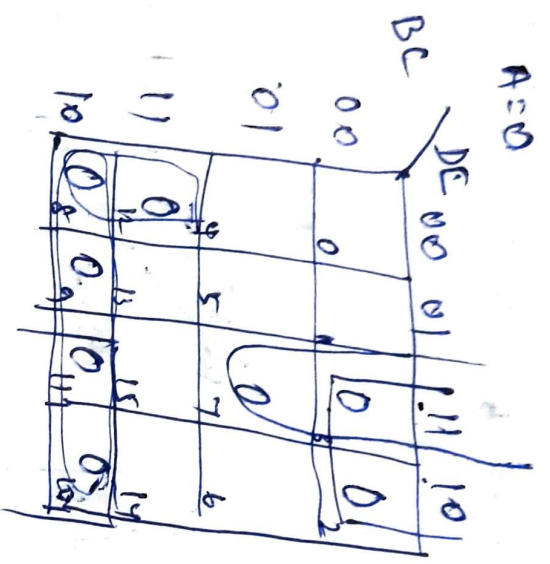
$f_{min} = \text{I} + \text{II} + \text{III}$   
 $\Rightarrow \bar{C} + A\bar{B} + A\bar{B}$



5 variables POS

$$F = \prod M(2, 3, 7, 8, 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 23, 26)$$

Ex.



$$m_{2,3, 19, 18, 11, 10, 24, 24} \rightarrow \bar{c}D = (C + \bar{D})$$

$$m_{16, 17, 20, 21} \rightarrow A\bar{B}\bar{D} = (\bar{A} + B + D)$$

$$m_{8, 9, 11, 10} \rightarrow \bar{A}B\bar{C} = (A + \bar{B} + \bar{C})$$

$$m_{1, 7, 14, 13} \rightarrow \bar{A}B\bar{C} = (A + \bar{B} + \bar{C})$$

$$m_{8, 12} \rightarrow \bar{A}\bar{B}\bar{D}\bar{E} = (A + \bar{B} + D + \bar{E})$$

$$f = (C + \bar{D})(\bar{A} + B + D)(A + \bar{B} + C)(B + \bar{D} + \bar{E})$$

$$(A + \bar{B} + D + \bar{E})$$

logic diagram

Ex. ① minimize the expression using Kmap.

$$Y = \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C}$$

101
001
100
011
000

step ①: plot the Kmap according to the given expression.

		BC			
		00	01	11	10
A	0	1 <sub>0</sub>	1 <sub>1</sub>	1 <sub>3</sub>	0 <sub>2</sub>
	1	1 <sub>4</sub>	1 <sub>5</sub>	0 <sub>7</sub>	0 <sub>6</sub>

1. octet 12
2. quad 13
3. pair of 1's
4. isolated 1's

step ② no octet & group quad I's

step ③ Group pair of 1's

step ④ no isolated ones

step ⑤ All 1's grouped

step ⑥ final expression

$$Y = I + II$$

$$= \overline{B} + \overline{A}C$$

$$\therefore Y = \overline{B} + \overline{A}C$$

Ex: 2 minimize the expression

$$Y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}\overline{C}D + A\overline{B}\overline{C}\overline{D} + A\overline{B}\overline{C}D + \overline{A}\overline{B}C\overline{D}$$

0000
0101
1100
1101
1001
0010

① Plot the Kmap

② quad 1's grouped

③ pair 1's grouped

④ Isolated 1's grouped

$$Y = I + II + III$$

$$Y = \overline{B}\overline{C} + \overline{A}\overline{C}D + \overline{A}\overline{B}C\overline{D}$$

		CD			
		00	01	11	10
AB	00	0 <sub>0</sub>	0 <sub>1</sub>	0 <sub>2</sub>	1 <sub>3</sub>
	01	1 <sub>4</sub>	1 <sub>5</sub>	0 <sub>7</sub>	0 <sub>6</sub>
	11	1 <sub>12</sub>	1 <sub>13</sub>	0 <sub>15</sub>	0 <sub>14</sub>
	10	0 <sub>8</sub>	1 <sub>9</sub>	0 <sub>11</sub>	0 <sub>10</sub>

② Reduce the following function using Karnaugh map technique & Implement using basic gates.

$$f(A, B, C, D) = \bar{A}\bar{B}D + A\bar{B}\bar{C}D + \bar{A}BD + A\bar{B}C\bar{D}$$

Note: The given function is not in the standard SOP form. So it's converted into SOP form.

$$\begin{aligned} f(A, B, C, D) &= \bar{A}\bar{B}D(C+\bar{C}) + A\bar{B}\bar{C}\bar{D} + \bar{A}BD(C+\bar{C}) \\ &= \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}BCD + \\ &\quad \bar{A}B\bar{C}D + A\bar{B}C\bar{D} \end{aligned}$$

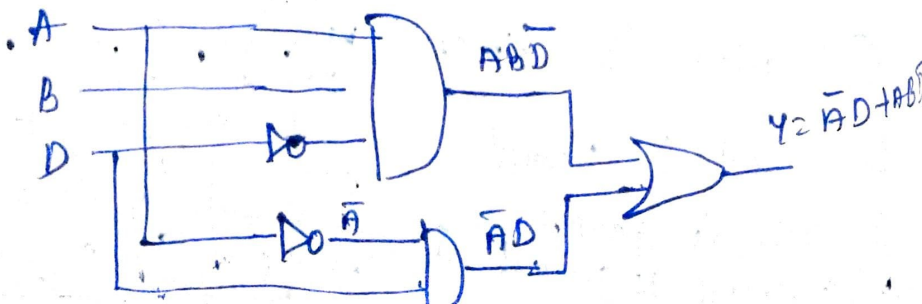
② plot the Kmap

		CD			
		00	01	11	10
AB	00	0	1	1	0
	01	0	1	1	0
	11	1	0	0	1
	10	0	0	0	0

Group 1:  $\bar{A}\bar{B}D$  (cells 1, 2, 3, 4)  
Group 2:  $\bar{A}BD$  (cells 12, 13, 14, 15)

$$\begin{aligned} Y &= \bar{I} + \bar{II} \\ &= \bar{A}D + A\bar{B}\bar{D} \end{aligned}$$

Implementation



Reduce the following function using K-map technique

$$f(A, B, C, D) = \sum m(0, 1, 4, 8, 9, 10)$$

① plot the kmap

		CD			
		00	01	11	10
AB	00	1	1	0	0
	01	1	0	0	0
	11				
	10	1	1		1

Groupings: I (00, 01), II (00, 01), III (10, 11), IV (10, 11)

$$Y = \overline{I} + \overline{II} + \overline{III}$$

$$f(A, B, C, D) = \overline{B} \overline{C} + \overline{A} \overline{C} \overline{D} + A \overline{B} \overline{D}$$

Don't care conditions

$$f(A, B, C, D) = \sum m(1, 3, 7, 11, 15) + \sum d(0, 2, 4)$$

		CD			
		00	01	11	10
AB	00	X	1	1	X
	01	X		1	
	11			1	
	10			1	

		CD			
		00	01	11	10
AB	00	1	1	1	1
	01			1	
	11			1	
	10			1	

$$Y = \overline{A} \overline{B} + CD$$

①  $f(A,B,C) = \sum m(0,1,3,7) + \sum d(2,5)$

	BC			
A	00	01	11	10
0	0	1	1	X
1	0	X	1	0

	BC			
A	00	01	11	10
0	1	1	1	1
1	0	1	1	0

$f(A,B,C) = \bar{A} + C$

② Find the reduced SOP form of following function.

$F(W,X,Y,Z) = \sum m(0,7,8,9,10,12) + \sum d(2,5,11)$

	YZ			
WX	00	01	11	10
00	1	0	0	X
01	0	X	1	0
11	1	X	0	0
10	1	1	0	1

$Y = \text{I} + \text{II} + \text{III}$

$= \bar{B}\bar{D} + A\bar{C} + \bar{A}BD = \bar{X}\bar{Z} + W\bar{Y} + \bar{W}XZ$

②

②



# Simplification of Product of Sums (POS) Expressions

$$(1) \quad Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C)(A+B+C)$$

 $M_1$  $M_3$  $M_7$  $M_4$  $M_0$ 

(i) Plot the K-map

(ii) Group the 0's

	BC	00	01	11	10
A	0	0	0	0	
1	0	0		0	

Groupings:  $\bar{I}$  (00),  $\bar{II}$  (01),  $\bar{III}$  (11),  $\bar{IV}$  (10)

$$\bar{Y} = \bar{I} + \bar{II} + \bar{III}$$

$$= \bar{B}\bar{C} + BC + \bar{A}C$$

$$Y = \overline{\bar{B}\bar{C} + BC + \bar{A}C}$$

Demorgan's law  $Y = (\overline{\bar{B}\bar{C}}) \cdot (\overline{BC}) \cdot (\overline{\bar{A}C})$

$$= (\bar{B} + \bar{C}) \cdot (\bar{B} + \bar{C}) \cdot (A + \bar{C})$$

$$= (\bar{B} + \bar{C}) (\bar{B} + \bar{C}) (A + \bar{C})$$

$$(2) \quad f(A, B, C, D) = \prod M(0, 2, 3, 8, 9, 12, 13, 15)$$

$$f = (\bar{A} + \bar{B} + \bar{D})(A + B + \bar{C})(\bar{A} + C)(A + B + D)$$

$$(2) \quad f(A, B, C, D) = \prod M(0, 3, 4, 7, 8, 10, 12, 14) + \sum d(2, 6)$$

AB	CD	00	01	11	10
00	0			0	X
01	0			0	X
11	0				0
10	0				0

$$\bar{f} = \overline{\bar{D} + \bar{A}C}$$

$$= D \cdot (\bar{A} + \bar{C})$$

$$f = (A + \bar{C}) D$$